# LO3 – Solve simultaneous linear equations using Gauss-Jordan elimination and matrices

Refresher:

Given 2x + 3y = 7 and -7x - 3y = 10

* Both represent lines; can plot to see where they intersect; can solve for x, y where they cross
* Possible result: cross at one point, parallel (no crossing), same line (always intersect)

Solved by:

1. Graph (rough point of intersection)
2. Add/subtract equations
3. Comparison
4. Substitutions (e.g. change 2x + 3y = 7 into 2x = 7 - 3y, then x = 7/2 - 3/2y, and then substitute for x into the other equation)

Fine for two unknowns; what if we have more equations and more unknowns?

Note: number of equations must match number of unknowns

Consider the following system of equations:

2*x* + 4*y* + 6*z* = 46

3*x* + 12*y* + 12*z* = 102

4*x* + 5*y* + *z* = 28

We would like to solve using a controlled process that can applied to *any* three equations.

This system of equations can be written as a matrix equation (multiply them out to see that they are the same):

Recall that in 3x = 12, 3 is the coefficient, x is the unknown, and 12 is the known value.

We can solve 3x = 12. Divide both sides by 3, so 1x = 4; 1 is the multiplicative identity for the set of integers, so that means that 1x = x = 4.

Goal: convert the coefficient matrix to identity matrix for matrix multiplication.

What can we do? There are three row operations that can be used to transform the coefficient matrix into the identity matrix:

1. Any two rows can be interchanged
2. Any row can be multiplied/divided by a non-zero constant
3. Any row can be multiplied/divided by a non-zero constant and added to another row

To simplify, write as an *augmented matrix* which is of size (*m*)×(*m*+1) with a square coefficient matrix and a column of known values (where *m* is the number of unknowns)

How can we convert this to the identity matrix?

* Start with the value in the upper-left corner (pivot value) – we want this to be one, so we divide the row by the pivot value
* Go down the first column to fill it with zeros so it looks like the identity matrix; so multiply the pivot row by a factor and add it to R2 and R3
  + R2=R2+R1\*-3
  + R3=R3+R1\*-4



Do this first

Do this first

Example 2:



*Verify:*

2(-3.4) + 3(4.6) = 7 [yes]

-7(-3.4) - 3(4.6) = 10 [yes]

**General solution strategy (ignore unsolvable equations for now)**

For each row

Reduce the pivot element to 1 (pivot is the element on the diagonal)

Use the pivot element to converts elements in the corresponding column to 0

**Algorithm**

For each row in the augmented matrix

pivot 🡨 get pivot for current row (pivot is on the diagonal)

For each element in the current pivot row

current element 🡨 current element / pivot

For each row in the matrix

If the current row is not the pivot row

dFactor = -1\*corresponding element to the pivot in the current row

For each column in the current row

current element 🡨 current element + dFactor \* corresponding element in pivot

**Practice:**

First practice:

x + y + 2z = 8

-1x - 2y + 3z = 1

3x - 7y + 4z = 10

Second practice:

2x + 2y + 4z = 12

2y + z = 4

2x + y + z = 7

Third practice: (unsolvable)

3x + 2y + 7z = 123

6x + 4y + z = 98

-3x - 2y + 8z = 10

**System solvable algorithm**

pivot 🡨 get pivot element for current row

while the pivot is 0

check rows below pivot for a non-zero pivot

if a non-zero pivot is found

swap pivot row with the non-zero row

else

indicate there is no solution

return pivot